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Existence Theorems of Two Families of Vector Generalized Quasi-Optimization Problems with Applications

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ABSTRACT

In this paper, we apply Himmelberg's fixed point theorem to establish existence theorems of two families of vector generalized quasi-optimization problems. We apply our results to establish existence theorems of systems of generalized vector-quasi-equilibrium problems. Systems of weak loose quasi-saddle point problem.

1 Introduction

Recently, Lin [7] considered simultaneous vector quasi-equilibrium problem and proved existence results for its solution. By using these results, he derived existence results for a solution of vector quasi-saddle point problem.

In the recent past, systems of scalar (vector) equilibrium problems, systems of scalar (vector) generalized equilibrium problems, systems of scalar (vector) quasi-equilibrium problems, and systems of scalar (vector) generalized quasi-equilibrium problems are used as tools to solve Nash equilibrium problem (for vector-valued functions) and Debreu type equilibrium problem(for vector-valued functions), respectively, see for example [1, 2, 3, 4, 5] and references therein.

Very recently, Ansari et al. [6] considered systems of simultaneous generalized vector quasi-equilibrium problem and proved existence results for its solution by scalarization method. By using these results, they derived existence existence results of a solution of system of vector quasi-saddle point problem.

Let I be any index set. For each $i \in I$, let E_i, V_i and Z_i be real locally convex topological vector spaces (in short, t.v.s.). For each $i \in I$, let $X_i \subset E_i$ be a nonempty convex set and $Y_i \subset V_i$ a nonempty convex set. Let $X = \prod_{i\in I} X_i$ and $Y = \prod_{i\in I} Y_i$. For each $i \in I$, let $S_i : X \times Y \multimap X_i$ be a multivalued map with nonempty values and $T_i : X \times Y \multimap Y_i$ be a multivalued map with nonempty values. Let $C_i : X \times Y \multimap Z_i$ be a multivalued map such that for each $(x, y) \in X \times Y$, $C_i(x, y)$ is a cone and $intC_i(x, y) \neq \emptyset$. Let $F_i : X \times Y \times X_i \multimap Z_i$ be a multivalued map with nonempty values and $G_i : X \times Y \times Y_i \multimap Z_i$ be a multivalued map with nonempty values.

Throughout this paper, we use these notation unless otherwise specified.

We first consider two families of vector generalized quasi-optimization problems :
Find a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, \bar{x}_i) \cap wMin_{C_i(\bar{x},\bar{y})} F_i(\bar{x}, \bar{y}, S_i(\bar{x}, \bar{y})) \neq \emptyset$ and $G_i(\bar{x}, \bar{y}, \bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} G_i(\bar{x}, \bar{y}, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

For the special case of above problems is systems of simultaneous generalized vector quasi-equilibrium problem for multivalued maps.
Find a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $F_i(\bar{x}, \bar{y}, x_i) \cap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x}, \bar{y})$ and $G_i(\bar{x}, \bar{y}, y_i) \cap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $y_i \in T_i(\bar{x}, \bar{y})$.
If F_i and G_i are single-valued maps. will be reduced to find a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $f_i(\bar{x}, \bar{y}, x_i) \notin (-intC_i(\bar{x}, \bar{y}))$ for all $x_i \in S_i(\bar{x}, \bar{y})$ and $g_i(\bar{x}, \bar{y}, y_i) \notin (-intC_i(\bar{x}, \bar{y}))$ for all $y_i \in T_i(\bar{x}, \bar{y})$.
This problem is a generalization of in Ansari et al. [5].

In section 4, we consider the following systems of weak loose quasi-saddle point problem.

Find $\bar{x} = (\bar{x}_i)_{i\in I} \in X$ and $\bar{y} = (\bar{y}_i)_{i\in I} \in Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $L_i(\bar{x}_i, \bar{y}_i) \cap wMax_{C_i(\bar{x},\bar{y})} L_i(S_i(\bar{x}, \bar{y}), \bar{y}_i) \neq \emptyset$ and $L_i(\bar{x}_i, \bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} L_i(\bar{x}_i, T_i(\bar{x}, \bar{y})) \neq \emptyset$, where $L_i : X_i \times Y_i \multimap Z_i$.

In this paper, we prove existence theorems of two families of vector generalized quasi-optimization problems by Himmelberg's fixed point theorem. Then we apply our results to study existence theorem of systems of weak loose quasi-saddle point problem and systems of generalized vector quasi-equilibrium problems. These results improved and generalized some main results in [5].

2 Preliminaries

Throughout this paper, all topological spaces are assumed to be Hausdorff.

Definition 2.1. Let Z be a real t.v.s., D a convex cone in Z with $intD \neq \emptyset$, and A a nonempty subset of Z. Let $y_1, y_2 \in A$, we denote $y_1 \leq y_2$, if $y_2 - y_1 \in D$; $y_1 < y_2$, if $y_2 - y_1 \in intD$.

A point $\bar{y} \in A$ is called a vector minimal point of A if for any $y \in A$, $y - \bar{y} \notin -D \setminus \{0\}$. A point $\bar{y} \in A$ is called a weakly vector minimal point of A if for any $y \in A$, $y - \bar{y} \notin -intD$. The set of vector minimal(resp. weakly vector minimal) points if A is denoted by $Min_D A$ (resp. $wMin_D A$).

3 Existence Results for a Solution of Two Families of Vector Generalized Quasi-Optimization Problems

Theorem 3.1. For each $i \in I$, let S_i be a continuous compact multivalued maps with nonempty closed convex values and T_i be a continuous compact multivalued maps with nonempty closed convex values. For each $i \in I$, assume the following conditions are satisfied :

(i) $C_i(x, y)$ is a closed convex pointed cone with apex at the origin and $intC_i(x, y) \neq \emptyset$;

(ii) the map $W_i : X \times Y \multimap Z_i$ defined by $W_i(x, y) = Z_i \setminus intC_i(x, y)$ is u.s.c. ;

(iii) F_i is a continuous multivalued map with nonempty compact values such that for any fixed $(x, y) \in X \times Y$, $F_i(x, y, u_i)$ is properly quasiconvex in u_i ; and

(iv) G_i is a continuous multivalued map with nonempty compact values such that for any fixed $(x,y) \in X \times Y$, $G_i(x,y,v_i)$ is properly quasiconvex in v_i.

Then there exists a $(\bar{x},\bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x},\bar{y})$, $\bar{y}_i \in T_i(\bar{x},\bar{y})$, $F_i(\bar{x},\bar{y},\bar{x}_i) \cap wMin_{C_i(\bar{x},\bar{y})}F_i(\bar{x},\bar{y},S_i(\bar{x},\bar{y})) \neq \emptyset$
and $G_i(\bar{x},\bar{y},\bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})}G_i(\bar{x},\bar{y},T_i(\bar{x},\bar{y})) \neq \emptyset$.

In particular, if for each $i \in I$, for all $x \in X$ and $y \in Y$, $F_i(x,y,x_i) \subset C_i(x,y)$ and $G_i(x,y,y_i) \subset C_i(x,y)$. Then there exists a $(\bar{x},\bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x},\bar{y})$, $\bar{y}_i \in T_i(\bar{x},\bar{y})$, $F_i(\bar{x},\bar{y},x_i) \bigcap (-intC_i(\bar{x},\bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x},\bar{y})$ and $G_i(\bar{x},\bar{y},y_i) \bigcap (-intC_i(\bar{x},\bar{y})) = \emptyset$ for all $y_i \in T_i(\bar{x},\bar{y})$.

Proof. For each $i \in I$, since S_i and T_i are compact, there exist compact subsets $D_i \subseteq X_i$ and $M_i \subset Y$ such that $S_i(X \times Y) \subseteq D_i$ and $T_i(X \times Y) \subseteq M_i$. For each $i \in I$ and for all $(x,y) \in X \times Y$, define two multivalued maps $\Phi_i : X \times Y \multimap D_i$ and $\Psi_i : X \times Y \multimap M_i$ by

$$\Phi_i(x,y) = \{u_i \in S_i(x,y) : F_i(x,y,u_i) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)) \neq \emptyset\}$$

and

$$\Psi_i(x,y) = \{v_i \in T_i(x,y) : G_i(x,y,v_i) \cap wMin_{C_i(x,y)}G_i(x,y,T_i(x,y)) \neq \emptyset\}.$$

Since $S_i : X \times Y \multimap X_i$ is a compact multivalued map with nonempty closed values, S_i has nonempty compact values. Since $F_i : X \times Y \times X_i \multimap Z_i$ is u.s.c. with compact values, $F_i(x,y,S_i(x,y))$ is a nonempty compact set for each $i \in I$, $\emptyset \neq Min_{C_i(x,y)}F_i(x,y,S_i(x,y)) \subset wMin_{C_i(x,y)}F_i(x,y,S_i(x,y))$.
Then there exists $k_i \in wMin_{C_i(x,y)}F_i(x,y,S_i(x,y))$ such that $k_i \in F_i(x,y,u_i)$ for some $u_i \in S_i(x,y)$. Therefore, $\Phi_i(x,y) \neq \emptyset$ for each $i \in I$ and for all $(x,y) \in X \times Y$. Suppose there exist some $(x,y) \in X \times Y$ and some $i \in I$ such that $\Phi_i(x,y)$ is not a convex subset of $S_i(x,y)$. Then there exist $v_i^1, v_i^2 \in \Phi_i(x,y)$ and $t \in [0,1]$ such that

$$tv_i^1 + (1-t)v_i^2 \notin \Phi_i(x,y). \tag{1}$$

We have $v_i^1 \in S_i(x,y)$, $v_i^2 \in S_i(x,y)$,

$$F_i(x,y,v_i^1) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)) \neq \emptyset.$$

and $F_i(x,y,v_i^2) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)) \neq \emptyset$.
Thus, there exists $a_i^1 \in F_i(x,y,v_i^1)$ such that for each $b_i \in F_i(x,y,S_i(x,y))$,

$$b_i - a_i^1 \notin -intC_i(x,y) \tag{2}$$

and there exists $a_i^2 \in F_i(x,y,v_i^2)$ such that for each $b_i \in F_i(x,y,S_i(x,y))$,

$$b_i - a_i^2 \notin -intC_i(x,y). \tag{3}$$

Since $S_i : X \times Y \multimap X_i$ is a multivalued map with nonempty convex values,

$$tv_i^1 + (1-t)v_i^2 \in S_i(x,y). \tag{4}$$

By (1) and (4), we have

$$F_i(x,y,tv_i^1 + (1-t)v_i^2) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)) = \emptyset. \tag{5}$$

Then for each $c \in F_i(x,y,tv_i^1 + (1-t)v_i^2)$, there exists $d_c \in F_i(x,y,S_i(x,y))$ such that $d_c - c \in -intC_i(x,y)$. (6)

By (2), (3) and conditions (iii), there exists $z_{a_i^1a_i^2} \in F_i(x,y,tv_i^1+(1-t)v_i^2)$ such that either $a_i^1 - z_{a_i^1a_i^2} \in C_i(x,y)$ (7)

or $a_i^2 - z_{a_i^1a_i^2} \in C_i(x,y)$. (8)

Without lost of generality, we may assume that (7) is true, then by (5), there exists $d_{z_{a_i^1a_i^2}} \in F_i(x,y,S_i(x,y))$ such that

$$d_{z_{a_i^1a_i^2}} - z_{a_i^1a_i^2} \in -intC_i(x,y). \tag{9}$$

By (7), $z_{a_i^1a_i^2} - a_i^1 \in -C_i(x,y)$ and (9), we have

$$\begin{aligned} d_{z_{a_i^1a_i^2}} - a_i^1 = (d_{z_{a_i^1a_i^2}} - z_{a_i^1a_i^2}) + (z_{a_i^1a_i^2} - a_i^1) &\in (-intC_i(x,y)) + (-C_i(x,y)) \\ &\subset -intC_i(x,y). \end{aligned} \tag{10}$$

By (2) and (10), we have a contraction. Therefore, for each $i \in I$ and for all $(x,y) \in X \times Y$, $\Phi_i(x,y)$ is a convex subset of $S_i(x,y)$.

For each $(x,y,u_i) \in \overline{Gr(\Phi_i)}$, there exists $(x^\alpha,y^\alpha,u_i^\alpha) \in Gr\Phi_i$ and $(x^\alpha,y^\alpha,u_i^\alpha) \to (x,y,u_i)$. One has $u_i^\alpha \in S_i(x^\alpha,y^\alpha)$ and

$$F_i(x^\alpha,y^\alpha,u_i^\alpha) \cap wMin_{C_i(x^\alpha,y^\alpha)}F_i(x^\alpha,y^\alpha,S_i(x^\alpha,y^\alpha)) \neq \emptyset. \tag{11}$$

Since $u_i^\alpha \in S_i(x^\alpha,y^\alpha)$ and S_i is u.s.c. with closed values, $u_i \in S_i(x,y)$. By (11), there exists $\{b_i^\alpha\}$ in Z_i such that

$$b_i^\alpha \in F_i(x^\alpha,y^\alpha,u_i^\alpha) \cap wMin_{C_i(x^\alpha,y^\alpha)}F_i(x^\alpha,y^\alpha,S_i(x^\alpha,y^\alpha)) \text{ for each } \alpha. \tag{12}$$

Let $K = \{(x^\alpha,y^\alpha,u_i^\alpha) : \alpha \in \Lambda\} \cup \{(x,y,u_i)\}$. Then K is a compact set. By conditions (iii), $F_i(K)$ is a compact set in Z_i. By (12), there exists a subnet $\{b_i^\beta\}$ of $\{b_i^\alpha\}$ such that $b_i^\beta \to b_i \in F_i(K)$.

Since $b_i^\beta \in F_i(x^\beta,y^\beta,u_i^\beta)$ and F_i is closed, $b_i \in F_i(x,y,u_i)$. Since $b_i^\beta \in F_i(x^\beta,y^\beta,u_i^\beta)$, $b_i \in F_i(x,y,u_i)$.

We need to show $b_i \in wMin_{C_i(x,y)}F_i(x,y,S_i(x,y))$.
For each $c_i \in F_i(x,y,S_i(x,y))$, we have $d_i \in S_i(x,y)$ such that

$$c_i \in F_i(x,y,d_i).$$

Since S_i is l.s.c. and $d_i \in S_i(x,y)$, there is a net $\{d_i^\beta\}$ such that $d_i^\beta \in S_i(x^\beta, y^\beta)$ and $d_i^\beta \to d_i$. Since F_i is l.s.c., and $c_i \in F_i(x,y,d_i)$, there is a net $\{c_i^\beta\}$ such that

$$c_i^\beta \in F_i(x^\beta, y^\beta, d_i^\beta) \text{ and } c_i^\beta \to c_i. \tag{13}$$

By (12) and (13), $c_i^\beta - b_i^\beta \notin -intC_i(x^\beta, y^\beta)$

$$\Leftrightarrow b_i^\beta - c_i^\beta \in Z_i \setminus intC_i(x^\beta, y^\beta) = W_i(x^\beta, y^\beta)$$

By condition (ii), W_i is a closed map and then $b_i - c_i \in W_i(x,y)$. Therefore, $c_i - b_i \notin (-intC_i(x,y))$ for all $c_i \in F_i(s,y,S_i(x,y))$ and

$$b_i \in wMin_{C_i(x,y)}F_i(x,y,S_i(x,y)). \tag{14}$$

By (14) and $b_i \in F_i(x,y,u_i)$, $b_i \in F_i(x,y,u_i) \cap wMin_{C_i(x,y)}F_i(x,y,S_i(x,y))$. Since $u_i \in S_i(x,y)$, $u_i \in \Phi_i(x,y)$ and $(x,y,u_i) \in Gr\Phi_i$. Therefore, $\Phi_i : X \times Y \multimap D_i$ is a closed map for each $i \in I$, it follows that Φ_i is u.s.c..

Since Φ_i is closed, $\Phi_i(x,y)$ is a closed set for each $(x,y) \in X \times Y$ and each $i \in I$. Similarly, for each $i \in I$, Ψ_i is u.s.c. and $\Psi(x,y)$ is a closed set for each $(x,y) \in X \times Y$ and each $i \in I$.

For each $i \in I$, define the multivalued map $A_i : X \times Y \multimap D_i \times M_i$ by

$$A_i(x,y) = (\Phi_i(x,y), \Psi_i(x,y)) \text{ for all } (x,y) \in X \times Y.$$

Then for each $i \in I$, A_i is u.s.c. with nonempty compact convex values. Let $D = \prod_{i\in I} D_i$ and $M = \prod_{i\in I} M_i$. The multivalued map $A : X \times Y \multimap D \times M$ defined by $A(x,y) = \prod_{i\in I} A_i(x,y)$ is u.s.c. with nonempty compact convex values. By Himmelberg fixed point theorem [6], there exists a point $(\bar{x},\bar{y}) \in D \times M$ such that $(\bar{x},\bar{y}) \in A(\bar{x},\bar{y})$. This means for each $i \in I$, $\bar{x}_i \in S_i(\bar{x},\bar{y})$, $\bar{y}_i \in T_i(\bar{x},\bar{y})$, $F_i(\bar{x},\bar{y},\bar{x}_i) \cap wMin_{C_i(\bar{x},\bar{y})}F_i(\bar{x},\bar{y},S_i(\bar{x},\bar{y})) \neq \emptyset$ and $G_i(\bar{x},\bar{y},\bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})}G_i(\bar{x},\bar{y},T_i(\bar{x},\bar{y})) \neq \emptyset$.
Then there exists $b \in F_i(\bar{x},\bar{y},\bar{x}_i)$ such that for each $c \in F_i(\bar{x},\bar{y},S_i(\bar{x},\bar{y}))$,

$$c - b \notin -intC_i(\bar{x},\bar{y})$$

If $F_i(x,y,x_i) \subseteq C_i(x,y)$, it is easy to see that Therefore, $F_i(\bar{x},\bar{y},x_i) \bigcap (-intC_i(\bar{x},\bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x},\bar{y})$ and $G_i(\bar{x},\bar{y},y_i) \bigcap (-intC_i(\bar{x},\bar{y})) = \emptyset$ for all $y_i \in T_i(\bar{x},\bar{y})$.

Remark 3.1. Theorem 3.1 is still true if condition (iii) is replaced by (iii)$'$ F_i is a continuous multivalued map with compact values and for any fixed $(x,y) \in X \times Y$, $F_i(x,y,u_i)$ is $C(x,y)$ quasiconvex in u_i.

With the same arguments as Theorem 3.1, we have the following theorem.

Theorem 3.2. In theorem 3.1, if the condition (iii) of Theorem 3.1 is replaced by

(iii$'$) $F_i : X \times Y \times X_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values such that for any fixed $(x,y) \in X \times Y$, $F_i(x,y,u_i)$ is properly quasiconcave in u_i.

Then there exists a $(\bar{x},\bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x},\bar{y})$, $\bar{y}_i \in T_i(\bar{x},\bar{y})$, $F_i(\bar{x},\bar{y},\bar{x}_i) \cap wMax_{C_i(\bar{x},\bar{y})} F_i(\bar{x},\bar{y},S_i(\bar{x},\bar{y})) \neq \emptyset$ and $G_i(\bar{x},\bar{y},\bar{y}_i) \cap wMin_{C_i(\bar{x},\bar{y})} G_i(\bar{x},\bar{y},T_i(\bar{x},\bar{y})) \neq \emptyset$.

Corollary 3.1. If conditions (iii) and (iv) of Theorem 3.1 is replaced by (iii)$'$ and (iv)$'$ respecetiy, where

(iii)$'$ $f_i : X \times Y \times X_i \to Z_i$ is a continuous function such that for all $x = (x_i)_{i \in I} \in X$ and $y \in Y$, $f_i(x,y,x_i) \in C_i(x,y)$ and for any fixed $(x,y) \in X \times Y$, the map $u_i \mapsto f_i(x,y,u_i)$ is properly quasiconvex.

(iv)$'$ $g_i : X \times Y \times Y_i \to Z_i$ is a continuous function such that for all $x \in X$ and $y = (y_i)_{i \in I} \in Y$, $g_i(x,y,y_i) \in C_i(x,y)$ and for any fixed $(x,y) \in X \times Y$, the map $v_i \mapsto g_i(x,y,v_i)$ is properly quasiconvex.

Then there exists a $(\bar{x},\bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x},\bar{y})$, $\bar{y}_i \in T_i(\bar{x},\bar{y})$, $f_i(\bar{x},\bar{y},x_i) \notin (-intC_i(\bar{x},\bar{y}))$ for all $x_i \in S_i(\bar{x},\bar{y})$ and $g_i(\bar{x},\bar{y},y_i) \notin (-intC_i(\bar{x},\bar{y}))$ for all $y_i \in T_i(\bar{x},\bar{y})$.

Corollary 3.2. In Theorem 3.1, if we assume that (i), (ii) and

(iii) $F_i : X \times Y \times X_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values such that for all $x \in X$ and $y \in Y$, $F_i(x,y,x_i) \subset C_i(x,y)$, and for any fixed $(x,y) \in X \times Y$, $F_i(x,y,u_i)$ is properly quasiconvex in u_i.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$ and $F_i(\bar{x}, \bar{y}, x_i) \bigcap (-intC_i(\bar{x}, \bar{y})) = \emptyset$ for all $x_i \in S_i(\bar{x}, \bar{y})$.

Corollary 3.3. In Theorem 3.1, if we assume (i) (ii) and

(iii) $G_i : X \times Y \times Y_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values such that for any fixed $(x, y) \in X \times Y$, $G_i(x, y, v_i)$ is properly quasiconvex in v_i.

Then there exists a $(\bar{x}, \bar{y}) \in X \times Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, and $G_i(\bar{x}, \bar{y}, \bar{y}_i) \cap wMin_{C_i(\bar{x}, \bar{y})} G_i(\bar{x}, \bar{y}, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

4 Applications to Systems of Loose Quasi-Saddle Point Problem and Constrained Competitive Nash-Type Equilibrium Problems

Theorem 4.1. Let I, F_i, V_i, Z_i, X_i, Y_i, X, Y, S_i and T_i be the same as in Theorem 3.1. Suppose that conditions (i), (ii) of theorem 3.1 are true. Suppose that

(iii) $L_i : X_i \times Y_i \multimap Z_i$ is a continuous multivalued map with nonempty compact values ;

  (a) for any fixed $y_i \in Y_i$, $L_i(x_i, y_i)$ is properly quasiconcave in x_i ; and

  (b) for any fixed $x_i \in X_i$, $L_i(x_i, y_i)$ is properly quasiconvex in y_i.

Then there exists a $\bar{x} = (\bar{x}_i)_{i \in I} \in X$ and $\bar{y} = (\bar{y}_i)_{i \in I} \in Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x}, \bar{y})$, $\bar{y}_i \in T_i(\bar{x}, \bar{y})$, $L_i(\bar{x}_i, \bar{y}_i) \cap wMax_{C_i(\bar{x}, \bar{y})} L_i(S_i(\bar{x}, \bar{y}), \bar{y}_i) \neq \emptyset$ and $L_i(\bar{x}_i, \bar{y}_i) \cap wMin_{C_i(\bar{x}, \bar{y})} L_i(\bar{x}_i, T_i(\bar{x}, \bar{y})) \neq \emptyset$.

Proof. For each $i \in I$, let $F_i(x, y, u_i) = L_i(u_i, y_i)$ and $G_i(x, y, v_i) = L_i(x_i, v_i)$.
Then Theorem 4.1 follows from Theorem 3.2.
If L_i is a single valued map, we have the following systems of vector quasi-saddle point problem.

Corollary 4.1. For each $i \in I$, let $S_i : X \multimap X_i$ be a continuous compact multivalued map with nonempty closed convex values and $T_i : Y \multimap Y_i$ be a continuous compact multivalued map with nonempty closed convex values. For each $i \in I$, assume the following conditions are satisfied.

(i) $C_i : X \multimap Z_i$ is a multivalued map such that for each $x \in X$, $C_i(x)$ is a closed convex pointed cone with apex at the origin and $intC_i(x) \neq \emptyset$;

(ii) the map $W_i : X \multimap Z_i$ defined by $W_i(x) = Z_i \setminus intC_i(x)$ is u.s.c. ;

(iii) $L_i : X_i \times Y_i \to Z_i$ is a continuous map such that

(a) for any fixed $y_i \in Y_i$, $L_i(x_i, y_i)$ is properly quasiconcave in x_i ; and

(b) for any fixed $x_i \in X_i$, $L_i(x_i, y_i)$ is properly quasiconvex in y_i.

Then there exists a $\bar{x} = (\bar{x}_i)_{i \in I} \in X$ and $\bar{y} = (\bar{y}_i)_{i \in I} \in Y$ such that for each $i \in I$, $\bar{x}_i \in S_i(\bar{x})$, $\bar{y}_i \in T_i(\bar{y})$, $L_i(\bar{x}_i, \bar{y}_i) - L_i(x_i, \bar{y}_i) \notin (-intC_i(\bar{x}))$ for all $x_i \in S_i(\bar{x})$. and $L_i(\bar{x}, y_i) - L_i(\bar{x}_i, \bar{y}_i) \notin (-intC_i(\bar{x}))$ for all $y_i \in T_i(\bar{y})$.

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